GROWTH RATES OF FUNCTIONS

In mathematics, it is often necessary to compare functions to determine how fast they grow. This can be done by determining the limit of the ratio of the two functions as the input goes to infinity. L'Hopital's rule is commonly useful in evaluating such a limit. The following table ranks successively faster-growing functions. Fill in the column on the right with the limits justifying the ranking.

Name	Function	Justification Using Limit of Ratio of Consecutive Functions
constant	1	$\lim_{n\to\infty} \frac{\ln\ln n}{1} = \ln\ln\infty = \infty$
double logarithmic	$\ln\ln n$	$\lim_{n\to\infty} \frac{\ln n}{\ln\ln n} =$
logarithmic	$\ln n$	
fractional power	n^p, $0 < p < 1$	
linear	n	
loglinear	$n \ln n$	
polynomial	n^p, $p > 1$	
exponential	e^n	
factorial	$n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$	
n to the n	$n^n = e^{n\ln n}$	
double exponential	2^{2^n}	

One use of this information is in evaluating the efficiency of algorithms. This is a major topic in computer science. "Big O" notation is used to measure how many operations an algorithm uses. Using a more efficient algorithm saves lots of time and money when the size of the input is large.

A closely related question is how quickly functions go to zero. Note that if $f(n) < g(n)$, then $\frac{1}{f(n)} > \frac{1}{g(n)}$. This is important for the comparison test for improper integrals and infinite series. It is also useful to determine how quickly infinite series converge. Complete the following chain of inequalities using the above functions (for n sufficiently large).

$1 > \frac{1}{\ln\ln n} > \frac{1}{\ln n} >$