BASICS OF CONIC SECTIONS

A conic section is a set of points in the plane that is formed by intersecting a cone and a plane. The equation of a cone with vertex at the origin that opens in the z dimension is $z^2 = a\left(x^2 + y^2\right)$. The four (non-degenerate) types of conic sections are the circle, ellipse, parabola, and hyperbola. Conic sections can also be defined geometrically as a set of points some distance(s) from a point and another point or line.

The general second degree two-variable equation is $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, with A, B, C not all 0. Specific classes of conic section can be specified with certain restrictions on the values of the constants $A - F$. When the conic section is centered at the origin and opens up/down or left/right, we have the standard form. The standard forms can also be written as parametric equations $(x, y) = (f(t), g(t))$. This information and some basic algebraic properties are presented in the table below.

	Circle	Ellipse	Parabola	Hyperbola
cone definition	plane perpendicular to cone axis	plane oblique to cone axis	plane parallel to side of cone	plane cuts both halves of cone
point/line definition	all points equidistant from a point (center)	all points whose distances from two points (foci) have a constant sum	all points equidistant from a point (focus) and line (directrix)	all points whose distances from two points (foci) have a constant difference
restrictions on $A - F$	$A = C,\ B = 0$	$B^2 - 4AC < 0$	$B^2 - 4AC = 0$	$B^2 - 4AC > 0$
standard form	$x^2 + y^2 = r^2$	$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1,\ a > b$	$y = ax^2$ or $x = ay^2$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
parametric equations	$(r\cos\theta, r\sin\theta)$	$(a\cos\theta, b\sin\theta)$	(t, at^2) or (at^2, t)	$(a\sec\theta, b\tan\theta)$
distance c from center/vertex to focus	0	$c^2 = a^2 - b^2$	$a = \frac{1}{4c}$	$c^2 = a^2 + b^2$
eccentricity	0	$0 < e = \frac{c}{a} = \frac{\sqrt{a^2-b^2}}{a} < 1$	1	$e = \frac{c}{a} = \frac{\sqrt{a^2+b^2}}{a} > 1$