

For functions involving vectors, there are two possibilities. The input can be an n -variable vector, and the output can be an m -variable vector.

Calculus I Concept	n -Variable Input	Vector Output (Length m)
function $y = f(x)$	function $f(\vec{x}) = f(x_1, \dots, x_n)$	vector-valued func. $\vec{f}(t) = (f_1(t), \dots, f_m(t))$
linear function $f(x) = kx$	linear function $f(\vec{x}) = \vec{a} \cdot \vec{x}$	linear transformation $T(\vec{x}) = A\vec{x}$, A matrix
definition of limit $L = \lim_{x \rightarrow a} f(x)$ $\forall \epsilon > 0, \exists \delta > 0$ such that if $0 < x - a < \delta$, then $ f(x) - L < \epsilon$	definition of limit $L = \lim_{\vec{x} \rightarrow \vec{a}} f(\vec{x})$ $\forall \epsilon > 0, \exists \delta > 0$ s.t. if $x \in B_\delta(a)$, then $ f(\vec{x}) - L < \epsilon$	vector-valued limit $\vec{L} = \lim_{t \rightarrow a} \vec{f}(t)$ $= (\lim_{t \rightarrow a} f_1(t), \dots, \lim_{t \rightarrow a} f_m(t))$ $\forall \epsilon > 0, \exists \delta > 0$ s.t. if $0 < t - a < \delta$, then $ \vec{f}(t) - \vec{L} < \epsilon$
various limit properties	analogous limit properties	
continuous at \vec{a} : $\lim_{x \rightarrow a} f(x) = f(a)$	$\lim_{\vec{x} \rightarrow \vec{a}} f(\vec{x}) = f(\vec{a})$	$\lim_{x \rightarrow a} \vec{f}(x) = \vec{f}(a)$
definition of derivative $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$	partial derivative $\frac{\partial f}{\partial x_i}(\vec{x}) = \lim_{h \rightarrow 0} \frac{f(x_1, \dots, x_i+h, \dots, x_n) - f(x_1, \dots, x_i, \dots, x_n)}{h}$	vector-valued derivative $f'(t)$ $= \lim_{h \rightarrow 0} \frac{\vec{f}(t+h) - \vec{f}(t)}{h} = (f'_1(t), \dots, f'_m(t))$
	gradient $\vec{\nabla} f(\vec{x}) = \left(\frac{\partial f}{\partial x_1}(\vec{x}), \dots, \frac{\partial f}{\partial x_n}(\vec{x}) \right) = Jf^T$	
	directional derivative (direction of \vec{u}) $D_{\vec{u}} f(\vec{a})$ $= \lim_{h \rightarrow 0} \frac{f(\vec{a} + h\vec{u}) - f(\vec{a})}{h} = \vec{\nabla} f(\vec{x}) \cdot \vec{u}$	definition of differentiable function f $\lim_{\vec{x} \rightarrow \vec{a}} \frac{ \vec{f}(x) - \vec{f}(a) - \vec{T}(\vec{x} - \vec{a}) }{ \vec{x} - \vec{a} } = 0$
	total derivative: linear trans. $T = Df(\vec{a})$ $T(\vec{x}) = (Df(\vec{a}))(\vec{x}) = \vec{\nabla} f(\vec{a}) \cdot \vec{x}$	total derivative: linear trans. $T = \vec{D}f(\vec{a})$ $T(\vec{x}) = (Df(\vec{a}))(\vec{x}) = \left[\frac{\partial f_i}{\partial x_j} \right] \vec{x}$
$\frac{d}{dx}(f + g) = \frac{df}{dx} + \frac{dg}{dx}$	$\vec{\nabla}(f + g) = \vec{\nabla}f + \vec{\nabla}g$	$\frac{d}{dx}(\vec{f} + \vec{g}) = \frac{d\vec{f}}{dx} + \frac{d\vec{g}}{dx}$
$\frac{d}{dx}(f \cdot g) = f \frac{dg}{dx} + g \frac{df}{dx}$	$\vec{\nabla}(f \cdot g) = f \vec{\nabla}g + g \vec{\nabla}f$	$\frac{d}{dx}(\vec{f} \cdot \vec{g}) = f \cdot \frac{d\vec{g}}{dx} + g \cdot \frac{d\vec{f}}{dx}$
		$\frac{d}{dx}(\vec{f} \times \vec{g}) = f \times \frac{d\vec{g}}{dx} + g \times \frac{d\vec{f}}{dx}$
chain rule $\frac{d}{dx}((g \circ f)(x)) = \frac{dg}{dx}(f(x)) \cdot \frac{df}{dx}(x)$	If $u = g(\vec{f}(t))$, $\frac{du}{dt} = \frac{\partial u}{\partial x_1} \frac{\partial x_1}{\partial t} + \dots + \frac{\partial u}{\partial x_n} \frac{\partial x_n}{\partial t}$	If $u = g(\vec{f}(s, t))$, $\frac{du}{ds} = \frac{\partial u}{\partial x_1} \frac{\partial x_1}{\partial s} + \dots + \frac{\partial u}{\partial x_n} \frac{\partial x_n}{\partial s}$
implicit differentiation If $f(x, y) = 0$, $\frac{dy}{dx} = -\frac{\partial f/\partial x}{\partial f/\partial y}$.	Implicit Function Theorem If $f(\vec{x}) = 0$, $\frac{dx_i}{dx_j} = -\frac{\partial f/\partial x_j}{\partial f/\partial x_i}$.	$\frac{du}{dt} = \frac{\partial u}{\partial x_1} \frac{\partial x_1}{\partial t} + \dots + \frac{\partial u}{\partial x_n} \frac{\partial x_n}{\partial t}$
antiderivative $\int f(x) dx$	see page 4	$\int \vec{f}(t) dt = \int f_1(t) dt + \dots + \int f_m(t) dt$