THEOREMS OF CALCULUS I

Calculus I contains many theorems. These theorems build on each other, so that later theorems are proved using earlier theorems, plus some key definitions. This theory can be hard to unravel when first taking calculus, so this sheet summarizes it. A few more difficult theorems are rarely proved in Calculus I; their proofs depend on results from advanced calculus.

In the following table, C means continuous, D means differentiable, and AD means antiderivative.

Theorem	**Statement**	**Proof Depends On**
Limit Laws	$\lim_{x\to a}(f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$, similar for other operations	$\epsilon - \delta$ definition of limit
Squeeze Theorem	If $f(x) \le g(x) \le h(x)$ near a, and $\lim f(x) = L = \lim h(x)$, then $\lim g(x) = L$.	$\epsilon - \delta$ definition of limit
Intermediate Value Theorem	If f is C on $[a,b]$ and $f(a) < L < f(b)$, $\exists\, c \in [a,b]$ with $f(c) = L$.	definition of continuous
Derivative Rules	too many to list	definition of derivative
Fermat's Theorem	If f has a local extremum at c, and $f'(c)$ exists, then $f'(c) = 0$.	definition of derivative
Extreme Value Theorem	If f is C on $[a,b]$, f has an absolute max and min on $[a,b]$.	Bolzano–Weierstrass Theorem
Rolle's Theorem	If f is D on $[a,b]$ and $f(a) = f(b)$, then $\exists\, c \in [a,b]$ with $f'(c) = 0$.	Extreme Value Theorem
Mean Value Theorem (MVT)	If f is D on $[a,b]$, then $\exists\, c \in [a,b]$ with $f'(c) = \frac{f(b)-f(a)}{b-a}$.	Rolle's Theorem
L'Hopital's Rule	If $f(a) = g(a) = 0$, and near a, f, g are D and $g' \neq 0$, $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$.	Cauchy's Mean Value Theorem
Zero Derivatives	If $f'(x) = 0$ on (a,b), then $f(x) = C$ on (a,b).	Mean Value Theorem
MVT for Definite Integrals	If f is C on $[a,b]$, $\exists\, c \in [a,b]$ with $f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$.	Intermediate Value Theorem
Existence of Integrals	If f is C on $[a,b]$, then f is integrable on $[a,b]$.	Extreme Value Theorem
Fundamental Theorem of Calculus I	If f is C on $[a,b]$, $A(x) = \int_a^x f(t)\,dt$ is an AD of f.	MVT for Definite Integrals
Fundamental Theorem of Calculus II	If f is integrable, and F is any AD of f on $[a,b]$, then $\int_a^b f(x)\,dx = F(b) - F(a)$.	Fundamental Theorem of Calculus I