USING THE LIMIT DEFINITION OF DERIVATIVE

The definition of the derivative function for $f(x)$ is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Example. $f(x) = x^2$.

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{h(2x+h)}{h} = \lim_{h \to 0} (2x+h) = 2x.$$

Determine the derivative of the following functions by using the definition.

1. $f(x) = x^3$.

2. $f(x) = \frac{1}{x}$.

3. $f(x) = \sqrt{x}$.

4. $f(x) = \frac{1}{x^2}$.

5. $f(x) = \frac{1}{\sqrt{x}}$.