Errata for Fundamentals of Graph Theory

July 25, 2021

This list contains corrections and clarifications for Fundamentals of Graph Theory. This includes discoveries that prove or refute conjectures mentioned in the text.

1. Basics of Graphs

(p. 16) Definition 1.39 should say "length of a shortest $u - v$ path".

(p. 26) The equation in Exercise 1.4.4 should be $\binom{n}{2} = \binom{k}{2} + k(n-k) + \binom{n-k}{2}$.

(p. 28) Exercise 1.5.24 should say "If G and $\overline{G}$ are both k-regular, what can be concluded about the order of G?"

(p. 30) Exercise 1.6.14 should say "Show that every closed walk of odd length contains an odd cycle."

(p. 32) Exercise 1.8.7 should be in Section 2.1.

2. Trees and Connectivity

(p. 43) Proposition 2.25 is due to Harary/Prins [1966].

(p. 45) This sentence is in the wrong place: "Harary graphs are named after Frank Harary, an early promoter of graph theory and author of several books on the subject."

(p. 58) Exercise 2.3.26 is due to Chartrand/Harary [1968].

3. Structure and Degrees

(p. 75) Theorem 3.20 is due to Fulkerson et al [1965] and Berge [1973].

(p. 85) The reference for Theorem 3.45 should be Dirac [1960].

(p. 97) Exercise 3.4.26a is due to Flores Pacheco et al [2001], Franceschini et al [2006].

4. Vertex Coloring

(p. 109) Proposition 4.16 is also due to Aberth [1964] and Behzad/Mahmoodian [1969].

(p. 111) Explicit constructions for k-chromatic graphs with girth g were later found (Lovasz [1968]).

(p. 113) The proof of Theorem 4.26 is due to Erdos [1970].

(p. 115) "An **interval graph** is ..."

(p. 123) Exercise 4.2.18 is due to Bean [1976].

(p. 125) Hedetniemi's [1966] conjecture that $\chi(G \times H) = \min\{\chi(G), \chi(H)\}$ was refuted by Shitov [2019]. He proved the existence of a pair of counterexamples with orders around 4^{100} and 4^{10000}.

(p. 126) Exercise 4.4.24c is due to Dmitriev [1986], Shaoji [1990], Patil [1991].

(p. 128) Exercise 4.5.25 is due to Borowiecki/Patil [1986].

(p. 129) Exercise 4.5.26 is due to Rose [1974].

(p. 129) Exercise 4.5.27 is due to Patil [1991].

(p. 129) Exercise 4.5.28 is due to Rose [1974] and Patil [1986].

5. Planarity

(p. 159) Exercise 5.2.22 is due to Franklin [1922].

(p. 161) Exercise 5.3.10 is due to Behzad/Mahmoodian [1969].

(p. 161) Exercise 5.3.11 is too hard. The Theorem, due to Harary/Karp/Tutte [1967] is:

A graph G has a planar square if and only if $d(v) \leq 3$ for all vertices v, every block of G with more than four vertices is an even cycle, and G does not have three mutually adjacent cut vertices.

6. Hamiltonian Graphs

(p. 177) The cycle A,D,B,F,E,C,A has total weight 42.

(p. 192) For clarity, exercises 11, 12, and 21 in 6.2 should refer to weighted graphs.

(p. 192) The weighted graphs referenced in Exercise 6.2.21 do not satisfy the triangle inequality, but Christofides' Algorithm can still be applied.

(p. 192) Exercise 6.3.4 is due to Proskurowski [1979].

7. Matchings

(p. 214) Theorem 7.44 when $\delta(G) \geq 2$ should be qualified "(if G is connected and $n > 7$)". The result is due to Blank [1973] and McQuaig/Shepherd [1989].

(p. 215) Algorithm 7.48 is due to Cockayne et al [1975]. See also Exercise 7.5.10 (p. 224).

(p. 217) There are two other graphs with $\delta(G) \geq 2$ and $\gamma_t(G) > \frac{4}{7}n$ formed from C_{10} by adding chords.

(p. 221) Exercise 7.3.12 is due to Mahmoodian [1981].

(p. 222) double (a)

(p. 222) Exercise 7.4.2 should say "Show that any uniquely 3-edge-colorable cubic graph has exactly three Hamiltonian cycles."

(p. 226) A clearer definition is that the **domatic number** $dom(G)$ of a graph G is the maximum number of dominating sets that partition the vertex set of G.

8. Generalized Graph Colorings

9. Decompositions

(p. 246) Definition 9.2 should be "A decomposition of K_n into copies of G is a **cyclic decomposition** if there is a cyclic permutation of the vertices that induces a permutation of the factors." (The definition given is that of a color-cyclic decomposition.)

(p. 247) Ringel's Conjecture has been proved for sufficiently large n (Montgomery et al [2020]).

(p. 254) The reference for Nash-Williams Theorem should be (Nash-Williams [1964]).

(p. 255) The conjecture of Hajos should say "Every graph with order n and all even degrees decomposes into at most $\left\lfloor \frac{n}{2} \right\rfloor$ cycles."

(p. 271) In Exercise 9.2.5, the requirement that the graph be bridgeless is unnecessary.

(p. 272) More generally, the **Linear Arboricity Conjecture** (Akiyama/Exoo/Harary [1980]) asserts that $la(G) \leq \left\lceil \frac{\Delta(G)+1}{2} \right\rceil$ for any graph G.

10. Appendices

(p. 280) The definition of distance should say "length of a shortest $u - v$ path".

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